High Reynolds Aerothermal Simulations and Reduced Basis in Feel++

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We present in this talk our work on model order reduction for aerothermal simulations. The model involves the resolution of coupled non-linear parametrized partial differential equations in which affine decomposition is not obtained. We consider the coupling between the incompressible Navier-Stokes equations and an advection diffusion equation for the temperature. This coupling can be either in one or two ways depending if we do consider only forced or both natural and forced convections.

Since the physical parameters induce high Reynolds and Peclet numbers, we have to introduce stabilisation operators in the formulation in order to deal with the well known numerical stability issue. The chosen stabilization, applied to both fluid and heat equations, is the usual Streamline-Upwind/Petrov-Galerkin (SUPG) which add artificial diffusivity in the direction of the convection field. However this method often produce non physical undershoots or overshoots in the edge of discontinuities, which can be critical when you want to ensure, for instance, the positiveness of certain fields.

To tackle this discontinuity problem, we add in our model a new operator, known in the literature as shock capturing method. This new operator is non-linear and adds artificial diffusivity in the region of the discontinuities in order to treat under/overshoots. Although this method is particularly efficient, it induces a new difficulty, because the system becomes fully non-linear.

We present in this talk our order reduction strategy for this model, based on Reduced Basis Method (RBM). In order to recover a affine decomposition for this complex model, we implemented a discrete variation of the Empirical Interpolation Method (EIM) [2, 3] which is a discrete version of original EIM. This variant allows to build a approximated affine decomposition for complex operators such as in the case of SUPG [1]. We also use this method for the non-linear operators induced by the shock capturing method.

The construction of a EIM basis for non-linear operators, involves a potentially huge numbers of non-linear FEM resolutions - depending of the size of the sampling. Even if this basis is built during an offline phase, we usually can not afford such expensive computational cost. We took advantage of the resent development of the Simultaneous EIM Reduced basis algorithm (SER) [4] to tackle this issue. Enjoying the efficiency offered by reduced basis approximation, this method provides a huge computational gain and can require as little as $N+1$ finite element solves where N is the dimension of the RB approximation. As an illustration we present a application of a cooling system of a printed circuit board with different heat sources. The model is parametrized with different physical and geometrical parameters.

This SER variant is now available in the generic and seamlessly parallel reduced basis framework of the opensource library Feel++ (Finite Element method Embedded Language in C++, http://www.feelpp.org). This work has been founded by the ANR project CHORUS.

Références

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